The H.N.S.B.Ltd. Science College, Himatnagar

Internal Examination November-2017

B.Sc. Semester: I | Subject: Mathematics | Date: 14 /11 /2017

Marks: 40 | Paper No.: CC-MAT-111 | Time: 11:30 to 1:00

Section -A

Answer any five of the following. (5)

1. $y = \frac{1}{ax+b} \Rightarrow y_n =$ _____.

   (A) $(-1)^n n!$ (B) $\frac{(-1)^n n!\, a^n}{(ax+b)}$ (c) $\frac{(-1)^n n! a^n}{(ax+b)^{n+1}}$

2. $\int_0^{\frac{\pi}{2}} cos^8 x\, dx =$ _____.

   (A) $\frac{35\pi}{256}$ (B) $\frac{7\pi}{256}$ (C) $\frac{5\pi}{256}$

3. $\int_0^{\frac{\pi}{2}} sin^5 x cos^3 x dx =$ _____.

   (A) $\frac{1}{42}$ (B) $\frac{1}{24}$ (C) None of this

4. If $\bar{x}$ and $\bar{y}$ are parallel then = _____.

   (A) $\bar{x} \times \bar{y} = \bar{o}$ (B) $\bar{x} \neq k\bar{y}$ (C) None of this

5. $\bar{a} \cdot (\bar{b} + \bar{c})$ = _____.

   (A) $\bar{a} \cdot \bar{b}$ (B) $\bar{a} \cdot \bar{c}$ (C) $\bar{a} \cdot \bar{b} + \bar{a} \cdot \bar{c}$

6. If $y = (ax + b)^m$ and n =m then yn = _____.

   (A) $n!$ (B) $n!\, a^n$ (C) $n!\, a^{n+1}$

7. $[\bar{a}\ \bar{b}\ \bar{c}]$ = _____.

   (A) $\bar{a} \cdot (\bar{b} \times \bar{c})$ (B) $\bar{a} \cdot (\bar{b} + \bar{c})$ (C) $(\bar{a} \times \bar{b}) \times \bar{c}$

Section –B

Answer any five of the following. (5)

8. If $y = \cos^{-1} x$ then find y_1.
9. If $f(x) = \log(1+e^x)$ then find $f'(0)$.
10. Find $\int_0^{\pi} sin^7 \frac{\theta}{2}\, d\theta$.
11. Find $\int_0^{\frac{\pi}{2}} sin^8 x\, cos^5 x\, dx$.
12. Find the limit for $\sum_1^{\infty} \frac{1}{2i+n}$
13. Write the condition of orthogonal sphere.
14. Given definition of reciprocal vectors.

Section –C

Answer any three of the following. (6)

15. If $y = \frac{2x+1}{(x-1)(2x-1)}$, $x \neq 1, \frac{1}{2}$ then find y_n.

16. Prove that $\int_0^1 \frac{x^5 cos^4(sin^{-1}x)}{\sqrt{1-x^2}} dx = \frac{8}{315}$.

17. Find the limit of series

$$\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{n+n} + \cdots$$

18. Find the volume of sphere with radius a.

19. *If* $\bar{a}, \bar{b}, \bar{c}$ *are any vector then prove that* $(\bar{a} + \bar{b}) \cdot (\bar{b} + \bar{c}) \times (\bar{c} + \bar{a}) = 2[\bar{a}\ \bar{b}\ \bar{c}]$.

Section -D

Answer any four of the following (12)

20. If $y = e^x \log x$ then prove $x y_{n+2} + (n+1-x) y_{n+1} - (n+1) y_n = e^x$, where $n > 1$.

21. Prove that $\lim_{n\to\alpha}[(1+\frac{1}{n})(1+\frac{2}{n})(1+\frac{3}{n}) \ldots (1+\frac{n}{n})]^{\frac{1}{n}} = \frac{4}{e}$.

22. Find the length of arc of the curve $3y^2 = (2x+8)^3$ between x=0 and x = 4 of first quadrant.

23. Find the surface area of right circular cone with radius r and height h.

24. Find out reciprocal vector set of the set $\{(1,1,-1),(1,-1,1),-1,1,1)\}$

25. Prove that $\int_0^{\sqrt{2}} x^5(2-x^2)^{\frac{5}{2}} dx = \frac{256\sqrt{2}}{693}$.

Section -E

Answer any two of the following (12)

26. State and prove Cauchy's mean value theorem.

27. Find the formula of $\int_0^{\frac{\pi}{2}} sin^n x\, dx$, $n \in N$.

28. Prove that

$(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = [\bar{a}\ \bar{c}\ \bar{d}]\bar{b} - [\bar{b}\ \bar{c}\ \bar{d}]\bar{a} = [\bar{a}\ \bar{b}\ \bar{d}]\bar{c} - [\bar{a}\ \bar{b}\ \bar{c}]\bar{d}$

29. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ then prove that $\nabla|\bar{r}| = \frac{\bar{r}}{|\bar{r}|}$ or $\nabla r = \frac{\bar{r}}{r}$, where $r = |\bar{r}|$.