The H.N.S.B.Ltd. Science College, Himatnagar

Internal Examination October-2016

B.Sc. Semester: I | Subject: Mathematics | Date: 05 /10 /2016

Marks: 40 | Paper No.: CC-MAT-111 | Time : 11:30 to 1:00

Section -A

Answer any five of the following. (5)

1. If $y = \log(1-x)$ then $y_n =$ _____.

   (A) $\frac{(-1)^{2n-1}(2n-1)!}{(1-x)^n}$ (B) $\frac{(-1)^{n-1}(n-1)!}{(1-x)^n}$ (c) $\frac{(-1)^n n!}{(1-x)^{n+1}}$

2. $\int_0^{\pi} \sin^3\frac{\theta}{2}\,d\theta =$ _____.

   (A) $\frac{3}{4}$ (B) $\frac{4\pi}{3}$ (C) $\frac{4}{3}$

3. $\int_0^{\frac{\pi}{2}} \sin^4 x\cos^6 x\,dx =$ _____.

   (A) $\frac{3\pi}{512}$ (B) $\frac{512\pi}{3}$ (C) $\frac{45\pi}{512}$

4. If $y = e^x \sin 2x$ then $y_n =$ _____.

   (A) $5^n \sin(2x + n\tan ½)$ (B) $(5)^{n/2}\sin(2x + n\tan^{-1}2)$

   (C) $5^n \sin 2x$

5. $\bar{a}, \bar{b}, \bar{c}$ are co-planner then $\bar{b} =$ _____.

   (A) $m_1\bar{a} - m_2\bar{c}$ (B) $m_1\bar{a} + m_2\bar{c}$ (C) $m_1\bar{a} + m_1\bar{c}$

6. $|\bar{a}|^2\,|\bar{b}|^2 - |\bar{a}\,\bar{b}|^2 =$ _____.

   (A) $|\bar{a}\times\bar{b}|$ (B) $\bar{0}$ (C) $|\bar{a}\times\bar{b}|^2$

7. Centre of sphere $x^2+y^2+z^2 - ax - by - cz = 0$ is _____.

   (A) $(\frac{a}{2},\frac{b}{2},\frac{c}{2})$ (B) $(\frac{-a}{2},\frac{-b}{2},\frac{-c}{2})$ (C) (a, b, c)

Section –B

Answer any five of the following. (5)

8. State Maclaurians theorem.
9. In R^2, Equation $x^2 - y^2 = a^2$. Convert to polar co ordinates.
10. Find the limit for

$$\frac{1}{n}\sum_{i=1}^{n}\sec^2\frac{i\pi}{4n}$$

11. Find the value of $\int_0^{\frac{\pi}{2}} \cos^8 x \, dx$.

12. Prove that div (Curl F) = 0.

13. Write the condition of orthogonal sphere.

14. In R^2, polar co-ordinate $(-\sqrt{2}, \frac{\pi}{4})$ convert to the Cartesian co-ordinate.

Section –C

Answer any three of the following. (6)

15. If $y = \frac{x}{x^2+2x-8}$, $x \neq 2,-4$ then find y_n.

16. Evaluate $\int_0^1 x^4 (1-x^2)^{\frac{7}{2}} dx$.

17. Prove that $(\bar{a} \times \bar{b}) \cdot (\bar{c} \times \bar{d}) = (\bar{a} \cdot \bar{c})(\bar{b} \cdot \bar{d}) - (\bar{a} \cdot \bar{d})(\bar{b} \cdot \bar{c})$.

18. If $\phi(x, y, z) = 2xz^2 - 3xy - 4x$ then find grad ϕ at point (1 ,-1 ,2).

19. Expand e^x, where $x \in R$.

Section -D

Answer any four of the following (12)

20. If $y = (x + \sqrt{x^2+1})^m$ then prove that
$(1+x^2) y_{n+2} + (2n+1) x y_{n+1} + (n^2 - m^2) y_n = 0$.

21. If $I_n = \frac{d^n}{dx^n}(x^n \log x)$ then prove that $I_n = n I_{n-1} + (n-1)!$ and also prove $I_n = n! \left[\log x + 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right]$

22. Find the limit for $[(1+\frac{1}{n})(1+\frac{2}{n})(1+\frac{3}{n}) \ldots (1+\frac{n}{n})]^{\frac{1}{n}}$.

23. Find out reciprocal vector set of the set $\{(4,1,2), (2,-1,1), (1,-1,1)\}$

24. If plane 2x -3y +z = K touches sphere $x^2 + y^2 + z^2 = 7$ then find k and co-ordinate of contact point.

25. Find the surface area of right circular cone with radius r and height h.

Section -E

Answer any two of the following (12)

26. State and prove Leibnitz's theorem.

27. Find the formula of $\int_0^{\frac{\pi}{2}} \cos^n x \, dx$, $n \in N$, $N>1$.

28. Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$,
where $r = |\bar{r}|$ and $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$.

29. Obtain the condition for the Plane lx +my +nz = p touches the sphere $x^2 + y^2 + z^2 = a^2$ and find the point of contact.